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Parton interactions in the Bjorken limit of QCD [1]

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Abstract:

We consider the Bjorken limit in the framework of the effective action approach and discuss its similarities to the Regge limit. The proposed effective action allows for a rather simple calculation of the known evolution kernels. We represent the result in terms of two-parton interaction operators involving gluon and quark operators depending on light-ray position and helicity and analyze their symmetry properties.

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1 Introduction

During three decades high energy processes related to the Bjorken limit of scattering amplitudes are playing a major role in the investigation of the hadronic structure and the interaction of hadronic constituents. Quite a number of theoretical concepts and methods have been developed resulting by now in a standard treatment described in textbooks [1] and reviews [2].

In the last years increasing attention is devoted to topics going beyond the standard deep inelastic structure functions, among them the long standing problem of the explicite treatment of higher twist contributions and their evolution [3], the generalization to non-forward kinematics [4], interpolating between the DGLAP [5] and the ERBL [6] limiting cases. The existence of this interpolation has been pointed out early in [7].

The small x behaviour of structure functions and related questions have drawn the attention also to the relations between the Bjorken and the Regge limits. The evolution in the latter asymptotics, as far as it proceeds in the perturbative region, is represented by the BFKL equation [8, 9, 10].

In view of these topics a discussion of the Bjorken asymptotics invoking concepts not aligned to the standard approach may be of interest.

We work out the effective action concept in analogy to the high energy effective action proposed to investigate the Regge asymptotics [11]. This allows to emphasize interesting similarities. We follow also the concepts developed in [12]. The basic idea is to formulate the factorization of short and large distance contributions in terms of amplitudes and to determine the multi-parton t-channel intermediate state. The partons represent here certain modes of the underlying gluon and quark fields. The operator product expansion is not the main tool in this formulation; however it leads to a natural and simple basis of operators, the essential part of which are the quasi-partonic operators. The scale dependence of the renormalized operators appears due to the interactions of the exchanged partons, being pair interactions in leading order represented by the so-called non-forward evolution kernels.

The effective action is obtained from the QCD action by separating modes and integrating over some modes. This action allows to obtain the parton interaction kernels by a simple calculation. We represent the interaction by hamiltonians involving (in leading order) two annihilation and two creation operators of partons. The latter depend on the parton type (gluon, quark flavour and chirality), parton helicity and on the position on the light ray. The hamiltonian form is convenient for investigating the symmetries of the exchanged multi-parton system. In particular this formulation is intended to provide a framework to analyze the integrability of the interaction.

The light ray position as the essential variable of operators in the Bjorken limit appeared explicitely first in the approaches by Geyer, Robaschik et al. [7] and by Balitsky and Braun [13].

The parton interaction operators can be obtained alternatively as external field effective vertices, where two type of external fields are introduces describing the asymptotic interaction with the currents of high virtuality and with the hadrons. This calculation can be done analyzing the space-time field configurations only without transforming to momentum space.

From the very beginning conformal symmetry was underlying the ideas about Bjorken scaling and its violation. It turned into a tool for finding multiplicatively renormalized operators [6, 14] and for relating the forward to the non-forward evolutions [12, 15]. Combining conformal and supersymmetry leads to interesting relations between different evolution kernels [12]. The non-forward evolution kernels have been reanalysed recently in [16]. Their reconstruction on the two-loop level from the results for the forward case has been considered in [17].

Integrability of the effective interaction in high energy QCD amplitudes has been discovered by Lipatov [18] in the Regge limit in the case of multiple exchange of reggeized gluons and a similar symmetry property was expected in the Bjorken limit. The first examples of higher twist evolutions tractable by integrability have been considered by Braun, Korchemsky et al. [19]. Some implications have also been considered in [20].

Discussing the Bjorken limit it may be convenient to have the amplitude of a definite process in mind. The non-forward deeply virtual Compton scattering off the proton , $\gamma^*(q_1) + p(p_1) \to \gamma^*(q_2) + p(p_2)$, is an appropriate example (Fig. 1a):

$$q_{1/2} = q' - x_{1/2}p', \quad q'^2 = 0, p'^2 = 0, p' \approx p_1, s = 2p'q', \qquad -q_{1/2}^2 = Q_{1/2}^2 = x_{1/2} \;\; s. \tag{1.1}$$

The Bjorken limit corresponds to $s \to \infty$ with the Bjorken variables x_1 and/or x_2 being of order 1.

This process is generic in the sense that already the two-parton exchange contribution has the kinematics of non-vanishing longitudinal momentum transfer which is encountered anyway in the subchannels of multi-parton exchange (higher twist) contributions. It is generic also because one limiting case, $x_1 = x_2$, is directly related to ordinary deep inelastic scattering and another limiting case, $x_1 = 0, x_2 = 1$, can be related by modifying the hadronic initial and final states to the factorized matrix elements (light-cone wave functions, distribution amplitudes) appearing in hard exclusive production.

2 Effective QCD action

2.1 Light-cone action

The Bjorken asymptotics of the amplitude is a sum over factorized terms, where one factor (A) is determined by the large momentum scale Q and the other (B) by the hadronic momentum scale m, Fig. 1b. The t-channel intermediate states of partons should be understood and defined in such a way that the corresponding exchange contribution is the one of a (quasi) real multi-particle state to the unitarity relation continued from the physical region of the t-channel. In this case the two factors are (continued from mass shell) multi-particle amplitudes having the valuable properties of gauge invariance and analyticity.

The transverse momenta κ_i of the exchanged partons are all of the same order κ, $(q \gg \kappa \gg m)$. The parton intermediate states are to be specified further in such a way that the integration over κ_i results in $\ln \frac{Q}{m}$, attributing the non-logarithmic remainder to other (non-leading) exchanges.

The approach to the asymptotics can be visualized as a process of iterating this factorization. The next step is shown in Fig. 1c. The Green function H (not necesserily connected) describes the parton interaction. The effective action is to describe just this parton interaction.

In the lowest order of perturbative expansion there are only pair interactions. The partons can be identified as modes of the underlying gluon A and quark $f, \tilde{f}$ fields in the gauge $q'^\mu A_\mu = 0$ after integrating over the redundant field components $p'^\mu A_\mu$ and $p'^\mu \gamma_\mu \; \psi$.

We represent 4-vectors x^μ by their light cone components $x_\pm$ and a complex number involving the transverse components $x_\perp = x_1 + ix_2$. In the case of the gradient vector we change the notation in such a way to have $\partial_+ x_- = \partial_- x_+ = 1, \partial_\perp x = \partial_\perp^* x^* = 1$. In particular the complex

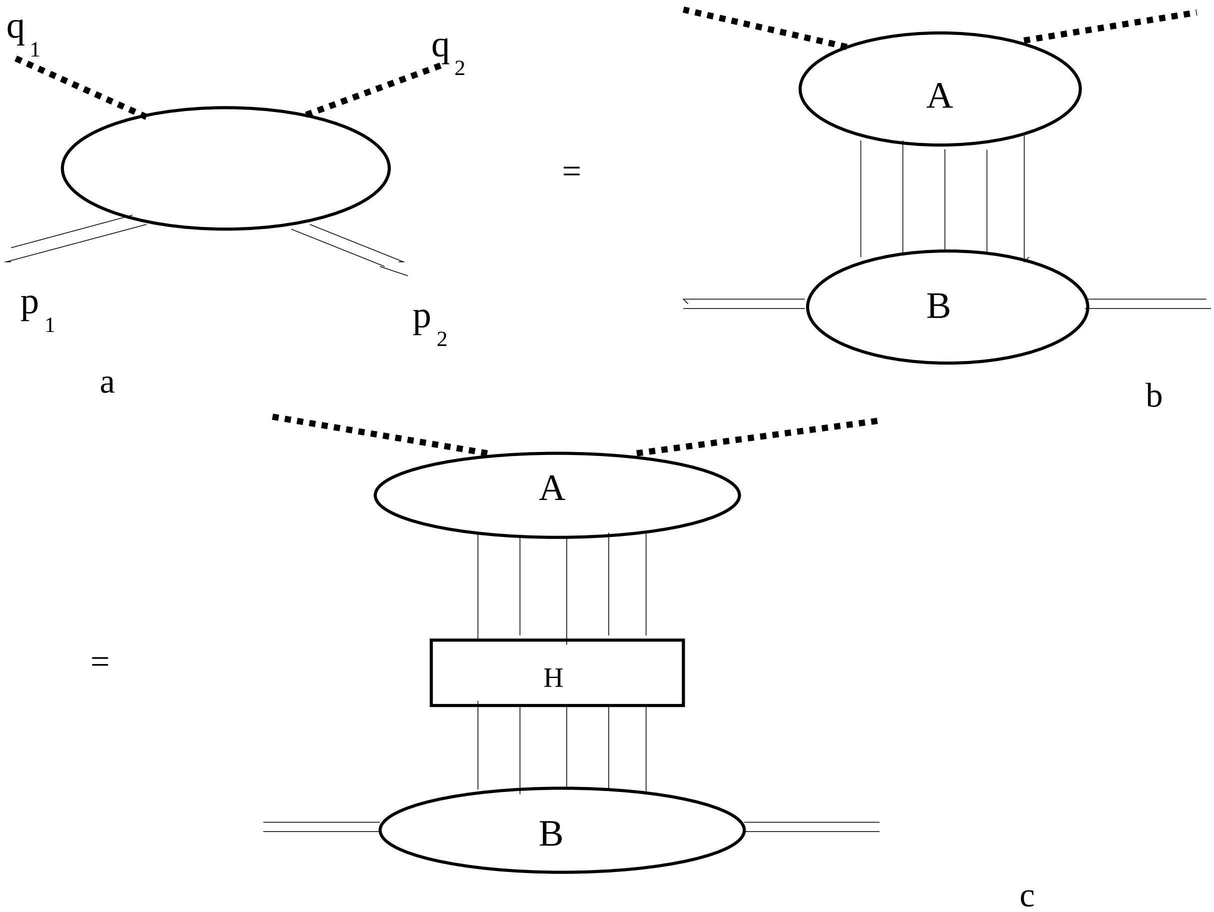

Figure 1: Large-scale factorization of the amplitude. A summation over t-channel intermediate states is implied.

valued field A represents the transverse components of the gauge field. We choose the frame where the light-like vector q' has the only non-vanishish component $q'_- = \sqrt{s/2}$ and p' the only non-vanishing component $p'_+ = \sqrt{s/2}$.

We decompose the Dirac fields into light cone components,

$$\psi = \psi_- + \psi_+, \quad \gamma_- \psi_+ = \gamma_+ \psi_- = 0$$
$$\psi_+ = f u_{+-} + \tilde{f} u_{++}, \quad \gamma^\mu = \frac{2}{s}(\gamma_- q'^\mu + \gamma_+ p'^\mu) + \gamma_\perp^\mu. \tag{2.1}$$

$u_{a,b}, \quad a, b = \pm$, is a basis of Majorana spinors,

$$\gamma_+ u_{-,b} = \gamma_- u_{+,b} = 0, \quad \gamma u_{a,-} = \gamma^* u_{a,+} = 0. \tag{2.2}$$

In terms of component fields $A, f, \tilde{f}$ the QCD action can be written as

$$\begin{aligned} S \;=\; \int d^4x \{ &-2A^{a*}(x)(\partial_+\partial_- - \partial_\perp \partial_\perp^*) A^a(x) + \\ & i f^{*\alpha}(x) \partial_+^{-1} (\partial_+\partial_- - \partial_\perp \partial_\perp^*) f^\alpha(x) + ... \\ \frac{g}{2}(\partial_1 \hat{V}_{123}^* [\partial_1 A^a(x_1)(A^*(x_2) T^a A^*(x_3)) &+ i f^\alpha(x_1)(f^*(x_2) t^\alpha A^*(x_3)) + ...]_{x_i = x} + \text{c.c.}) \end{aligned}$$

$$+\frac{g^2}{4}\hat{V}_{11',22'}[2(A^*(x_1)T^c\partial A(x_{1'})(\partial A^*(x_2)T^c A(x_{2'}))+$$
$$+\frac{1}{2}(f^*(x_1)t^c f(x_{1'}))(f^*(x_2)t^c f(x_{2'}))+i(f^*(x_1)t^c f(x_{1'}))(A^*(x_2)T^c \overleftrightarrow{\partial} A(x_{2'}))$$
$$-i(f^*(x_1)t^\alpha A(x_{1'}))\partial(A^*(x_2)t^{*\alpha}f(x_{2'}))+...]_{x_i=x'_i=x}\ \}. \tag{2.3}$$

The periods stand for terms obtained from the written ones by replacing pairs of fermion fields f^*, f of one chirality by the ones of the other chirality $\tilde{f}^*, \tilde{f}$ and by the pairs of other flavour fermion fields. The elimination of redundant field components has lead to non-local vertices,

$$\hat{V}^*_{123} = \frac{i}{3\partial_1\partial_2\partial_3}[\partial^*_{\perp 1}(\partial_2-\partial_3)+\text{cycl.}],\quad \hat{V}_{11',22'} = (\partial_1+\partial_{1'})^{-2}. \tag{2.4}$$

Here and in the following we omit the space index + on derivatives, i.e. derivative operators not carrying subscripts $-, \perp$ are to be read as ∂_+. Integer number subscripts refer to the space point on which the derivative acts. The definition of the inverse ∂^{-1} is to be specified by the Mandelstam-Leibbrandt prescription.

The gauge group structure of the action has been written by using brackets combining two fields into the colour states of the adjoint (a) and of the two fundametal (α and $*\alpha$) representations:

$$(A_1^*T^aA_2) = -if^{abc}A_1^{*b}\ A_2^c,\ (f_1^*t^af_2) = t^a_{\alpha\beta}f_1^{*\alpha}f_2^\beta,$$
$$(f^*t^\alpha A) = t^c_{\beta\alpha}f^{*\beta}A^c,\ (A^*t^{*\alpha}f) = t^b_{\alpha\beta}A^{*b}f^\beta. \tag{2.5}$$

2.2 Separation of momentum modes

The partons in the intermediate state between the high (low) momentum scale amplitude factor A (B) and H are the large (+) (small (-)) transverse momentum modes of the fields $A, f, \tilde{f}$. We have the mode decomposition

$$A = A^{(+)} + A^{(0)} + A^{(-)} \tag{2.6}$$

(and analogously for the other fields). The medium modes $A^{(0)}$ are not important in the leading contribution discussed below. Medium modes have to be separated in the leading contribution to the double logarithmic approximation, compare Sect. 4.

The bare effective action of the leading parton interaction in the Bjorken asymptotics is obtained from (2.3) by substituting the mode separation and simplifying the vertices by the approximation,

$$\partial^\perp A^{(-)}\ =\ 0,\quad \partial^\perp f^{(-)} = 0,\quad \partial^\perp \tilde{f}^{(-)} = 0. \tag{2.7}$$

In particular the terms involving two fields in (+) modes are the ones important for deriving the bare $2 \to 2$ parton interaction operators, as will be explained in an example in Sect. 3.

This effective action is the analogon of the high-energy effective action considered in [11] as a tool for investigating the Regge limit in QCD. Unlike the Regge case here the effective vertices are just the original QCD vertices reduced by the conditions (2.7). There is no analogon to the induced contribution to the triple vertices.

Going beyond the tree approximation one should notice that in this action the modes of virtualness essentially larger than the scale κ of the (+) modes have been integrated out. In particular the coupling is the one at scale κ, $\alpha_S(\kappa^2)$, and the the propagators of the (+) modes are the bare ones at this scale. With the running coupling one defines the evolution variable ξ,

$$\xi(\kappa^2, m^2) = \int_{m^2}^{\kappa^2}\frac{d\kappa'^2}{\kappa'^2}\ \frac{\alpha_S(\kappa^2)}{2\pi}. \tag{2.8}$$

The propagators of the (-) modes, however, are normalized to be the bare ones at the scale $\kappa', \kappa' \ll \kappa$, appearing in the next iteration. Therefore Z factors of renormalization

$$Z_p = \exp(-w_p \xi(\kappa^2, \kappa'^2)), \tag{2.9}$$

where p labels the parton type, $p = A$ for gluon and $p = f$ for quark, are to be included in the Green function H of parton interactions.

$$w_p = C_p \left(\int_0^1 \frac{d\beta}{1-\beta} \ + \ w_p^{(0)} \right), \qquad C_A = N, \quad C_f = \frac{N^2 - 1}{2N},$$
$$w_A^{(0)} \ = -\frac{1}{4}(\frac{11}{3} - \frac{2N_f}{3N}), \qquad w_f^{(0)} \ = -\frac{3}{4}. \tag{2.10}$$

The Z factors refer to a finite change of renormalization scale. The parton anomalous dimensions w_p are infrared divergent (appearing as an artifact of the gauge chosen). The divergence is written explicitely above without specifying a regularization.

Comparing to the Regge case one understands that the Z factors in the parton propagators are the analogon of the reggeization factors $(\frac{s}{m^2})^{\omega_p(\kappa)}$ in the reggeon propagators of gluons or quarks in the Regge case. The anomalous dimension w_p is the analogon of the trajectory function $\omega_p(\kappa)$. Also the point about the infrared divergencies and their cancellation is analogous: In the same way as in the Regge case the infrared divergencies in the bare interactions are compensated by the ones in w_p. The infrared cancellation in the multi-parton exchange channel of overall colour singlet holds by the same arguments as in the multi reggeon exchange [21]:

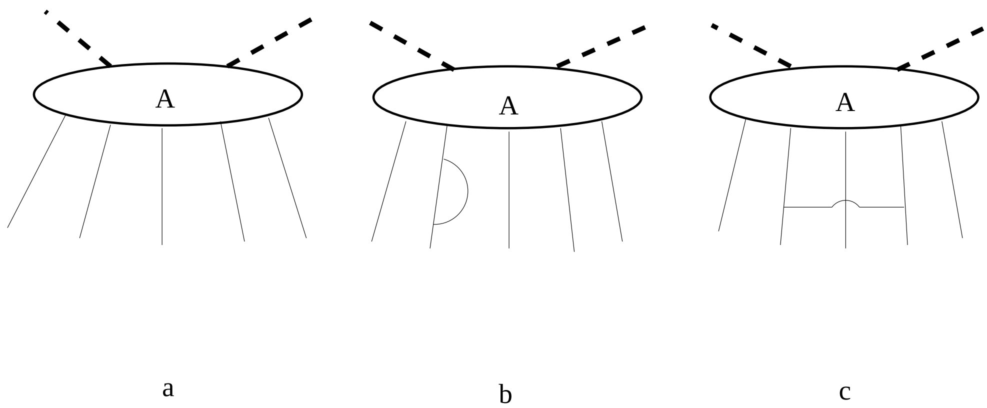

Figure 2: Compensation of infrared singularities.

The infrared singular contribution arising from vanishing gluon (longitudinal in the Bjorken case and transverse in the Regge case) momentum from the loop in Fig. 2c is twice the one in Fig. 2b, besides of the gauge group factors. The relation of these group factors follows from the observation that the exchanged partons or reggeons are in the overall colour singlet state, i.e. a gauge group transformation is acting on the group indices $a_1, ..., a_n$ (at this point the notation does not distingiuish between the adjoint (a) and fundamental (α) representations) of each of the factorized amplitudes as

$$\left(\exp[i \sum \varepsilon^a T_i^a]\right)_{a'_1,...,a'_n}^{a_1,...,a_n} A^{a'_1,...a'_n} \ = \ A^{a_1,...,a_n} \tag{2.11}$$

The generators $T_i^a, a = 1, ..., N^2-1$, act in the representation space of the parton or reggeon p_i. Expanding up to the second order in the group parameters ε^a we find that in acting on the considered amplitudes one can identify

$$\sum_i T_i^a \cdot T_i^a \;=\; -2 \sum_{i<j} T_i^a \otimes T_j^a, \tag{2.12}$$

where $T_i^a \cdot T_i^a = I_i \; C_{p_i}$, with I_i being the identity matrix and C_{p_i} the eigenvalue of the Casimir operator in the representation corresponding to the parton p_i.

This allows to rewrite the singular contributions of Fig. 2b as decomposed in terms of the same gauge group factors as appearing in Fig. 2c and to perform the cancellation of the infrared singularities explicitely.

The leading parton interaction is calculated by writing the Green function H according to the effective action and extracting the logarithmic contribution from the loop integrals of the $(+)$ mode intermediate state over the transverse momenta κ and the longitudinal (parallel to q' momenta α. These integrals are done in a standard way leaving one-dimensional (in the longitudinal momentum β parallel to p') loop integrals. In this way the parton interaction appears reduced to one dimension, in the light ray in space-time parallel to q') or in the light cone momentum (fraction) parallel to p'. Again this is analogous to the Regge case with an important difference: In the Regge case the leading contribution is obtained by extracting the logarithm from the longitudinal momentum integrations (α and β) and therefore the reggeon interaction is reduced to the two transverse dimensions.

It is well known that in the Regge case the most convenient way to deal with the trivial longitudinal dimensions is to change to the continued t-channel partial waves, i.e. to Mellin transform the amplitude factors with respect to energies. In the same way it turns out to be convenient to treat the trivial transverse momentum dependence of the Bjorken limit amplitude factors by a Laplace transform in ξ ($\xi(Q^2, \kappa^2)$ in A, $\xi(\kappa^2, \kappa'^2)$ in H , $\xi(\kappa'^2, m^2)$ in B). We perform this transformation after changing the momentum variable $\alpha = \frac{2(kp)}{s}$ to $\bar{\alpha} = \frac{\alpha s}{|\kappa|^2}$, because the integrals in $\bar{\alpha}$ are standard ones depending on momentum components $\beta = \frac{2(kq')}{s}$ only.

The multi-loop convolution of the factorized parts A, H, B of the amplitude reduces to a convolution in the longitudinal momentum fractions β only. The Laplace transforms of A, H, B enter at the same value of the variable ν Laplace conjugated to ξ. The dependence on ν of the leading contribution to the parton interaction H is just through the factor $\frac{1}{\nu}$,

$$H(\alpha_i, \beta_i, \kappa_i; \alpha'_j, \beta'_j, \kappa'_j) \to \frac{1}{\nu} H^{(0)}(\beta_i; \beta'_j) \tag{2.13}$$

The Z factor contribution may be included at this point by the replacement

$$\frac{1}{\nu} \to \frac{1}{\nu + \sum_i w_{p_i}} \tag{2.14}$$

The sum is over the exchanged partons of type p_i. This is the "energy denominator" ($-\nu$ being the analogon of E) attributed to the propagation of a multi-parton intermediate state between two subsequent interactions.

The iterated factorization described above can be formulated as an evolution equation in ξ for the lower scale amplitude factor B.

$$M_B(\beta_i) = M_B^{(0)}(\beta_i) +$$

$$\frac{1}{\nu + \sum w_p} \int d\beta' \delta(\sum \beta_i - \sum \beta'_j) \;\; H^{(0)}(\beta_i; \beta'_j) \;\; M_B(\beta'_j) \tag{2.15}$$

We have not written the gauge group and parton type indices carried by M_B and H and summed over in their contraction. The redefined kernels

$$H(\beta_i; \beta_j) = H^{(0)}(\beta_i; \beta'_j) + \prod_i^{n-1} \delta(\beta_i - \beta'_i) \sum_i^n w_{p_i} \tag{2.16}$$

represent infrared finite operators. The rearrangement of the gauge group factors explained above (2.12) is implicite here. The solutions of the evolution equation (2.15) is obtained in terms of a complete set of eigenfunctions of these operators and their eigenvalues. Usually one associates with the eigenfunctions composite operators, being renormalized multiplicatively, and calls the corresponding eigenvalues anomalous dimensions of these operators.

2.3 Space-time picture

The effective action describing the parton interactions in block H of the virtual Compton amplitude can be alternatively treated from the space-time point of view. Indeed, H describes the interaction between sources located in the vicinity of the light ray $x_\perp = 0, x_- = 0, x_+ = z \in \mathbf{R}^1$ and other sources the distribution of which is almost constant in the variables $x'_\perp, x'_+$ depending essentially only on the coordinate along the light-ray $x'_- = z'$. The small width of the first distribution near the light cone is characterized by the short distance scale $\Delta \sim Q^{-1}$ and the variation of the second distribution in directions away from the light ray is determined by the large distance scale $\delta \sim m^{-1}$.

Consider now the QCD functional integral with such sources or the related vertex functional with corresponding external gluon and quark fields. Instead of separating momentum modes (2.6) from this viewpoint we divide now the fields of quarks and gluons into two types of external fields $A^{(\pm)}$ and a quantum fluctuation A_q,

$$A \rightarrow A^{(+)} + A_q + A^{(-)}. \tag{2.17}$$

$A^{(+)}$ has the support near the light cone and has to be substituted by the following expression in terms of (regularized) delta functions,

$$A^{(+)}(x) = A_1(z)\delta(x_+)\delta^{(2)}(x_\perp) + \mathcal{O}(\Delta). \tag{2.18}$$

The other external field $A^{(-)}$ has to be substituted as

$$A^{(-)}(x') \;\; = \;\; A'_1(z') \text{ const} + \mathcal{O}(m). \tag{2.19}$$

The vertex functional or the external field effective action is now obtained by doing the integration over the quantum fluctuations A_q. Consider in particular the resulting vertex involving two $A^{(+)}$ and two $A^{(-)}$ type fields on the tree level. It has the form

$$\begin{aligned} &\int d^4x_1 d^4x_2 d^4x_{1'} d^4x_{'} A^{(+)}(x_1) A^{(+)}(x_2) G(x_{11'}) G(x_{22'}) \\ &[\tilde{V}_{11'} G(x_{1'2'}) \tilde{V}_{22'} + \delta^{(4)}(x_{1'2'}) V_{11',22'}] A^{(-)}(x_{1'}) A^{(-)}(x_{2'}) \end{aligned} \tag{2.20}$$

$\tilde{V}_{11'}, \tilde{V}_{22'}$ are simply related to the triple vertex in (2.3) depending on the case considered and $G(x)$ stands for the quark or gluon propagator. We substitute the particular asymptotic form of the external fields (2.18, 2.19) and obtain

$$\int dz_1 dz_2 dz_{1'} dz_{2'} A_1 A_2 A_{1'} A_{2'} \;\; c(\frac{\delta}{\Delta}) \;\; \left[\tilde{V}_{11'} \tilde{V}_{22'} \tilde{J}_{111} + V_{11',22'} \tilde{J}_0\right] \tag{2.21}$$

We abbreviate the residual dependence of the parton fields on the light ray positions by indices $1, 2$ for $A_1^{(+)}(z_1)$, $A_1^{(+)}(z_2)$ and by indices $1', 2'$ for $A_1^{(-)\prime}(z_{1'})$, $A_1^{(-)\prime}(z_{2'})$. The integration over the transverse and + components of x_1, x_2 is done due to the delta functions and the integrals over the transverse and + components of $x_{1'}, x_{2'}$ are represented by $\tilde{J}_{111}, \tilde{J}_0$,

$$\tilde{J}_{111} = c(\delta/\Delta)^{-1} \int dx_{1'-} dx_{2'-} d^2x_{1'\perp} d^2x_{2'\perp} \partial_1^\perp \partial_1^{\perp *} (x_{11'}^2)^{-1} (x_{22'}^2)^{-1} (x_{1'2'}^2)^{-1}$$
$$\tilde{J}_0 = c(\delta/\Delta)^{-1} \int dx_{1'-} dx_{2'-} d^2x_{1'\perp} d^2x_{2'\perp} \partial_1^\perp \partial_1^{\perp *} (x_{11'}^2)^{-1} (x_{22'}^2)^{-1} \delta^{(4)}(x_{1'2'}) \qquad (2.22)$$

The integrals are regularized by taking into account the smearing of the near light cone distribution by Δ and the large scale cutoff δ for the other distribution. We shall do the integration in the logarithmic approximation. For this we have substituted in (2.21) already the effective triple vertices omitting terms which do not result in logarithmic integrals.

In this approximation and after correcting the normalization by a factor $c(\delta/\Delta) \sim (\ln(\delta/\Delta))$ these integrals coincide with the Fourier transform of the standard J integrals to be defined below (5.4).

For going beyond the leading approximation one should specify accordingly in more detail the asymptotic space-time dependence of the external fields (2.18, 2.19), include loop corrections into the vertices and perform the integrals beyond the logarithmic approximation.

In the above equations we did not distinguish explicitely the cases of quarks and gluons and we have suppressed the colour and helicity structures. It turns out that these stuctures are the same for the J_{111} and the J_0 contributions. In this way the effective interaction vertices factorize right from the first step of calculation into one acting on the light ray positions and one acting on the helicity, chirality, colour and flavour degrees of freedom.

The operators acting on the positions arise as a sum of singular terms. The singularities cancel partially due to a relation between the J integrals involved. To cancel the remaining singularities and to arrive at well defined operators the disconnected contributions of one-loop self-energy corrections have to be included.

Let us point out that in this framework it is readily understood that in the approximation of leading twist and leading logarithms there are no vertices resulting in the change of the number of partons in the exchange channel. This is similar to the multi reggeon exchange. Consider for example the vertex of $A^{(+)}A^{(+)}A^{(-)}$. There is a logarithmic contribution in the integration over the coordinates besides of the light-ray ones; however in this term $A^{(-)}$ enters with a transverse derivative and this vanishes in the considered approximation (compare (2.19)).

We have outlined the calculation of the vertex of two-parton interaction to order g^2 in the logarithmic approximation. It is given by (2.21) up to the coefficient $\frac{g^2}{8\pi^2} \ln(\frac{\delta^2}{\Delta^2})$. By renormalization group the result can be extended to all orders in the leading logarithmic approximation. This amounts in replacing the kernel in this vertex (2.21) by the Green function of the Hamiltonian operator obtained from (2.21) by converting the external fields to operators, creation operators for the fields of type $A^{(+)}$ at points $1, 2$ and annihilation operators for the ones of type $A^{(-)}$ at $1', 2'$. The role of (euclidean) time in this Hamiltonian picture is played by the evolution variable $\xi(\delta^2, \Delta^2)$ (2.8) , related to the logarithmic coefficient of the above vertex by $d\xi = \frac{g^2(\kappa^2)}{8\pi^2} \frac{d\kappa^2}{\kappa^2}$, where the coupling $\frac{g^2}{4\pi}$ is replaced by the (one-loop) running coupling $\alpha_S(\kappa^2)$.

The spectrum of the Hamiltonian operator is actually the set of anomalous dimensions appearing in the considered channel.

3 Example of two-parton interaction

3.1 Energy-momentum representation

In order to illustrate the scheme described above we do the calculation for the lowest order interaction of two gluons of parallel helicities. The contribution of the triple vertex (2.3) reduced to the effective vertex by the condition (2.7) to the graphs Fig. 3 are

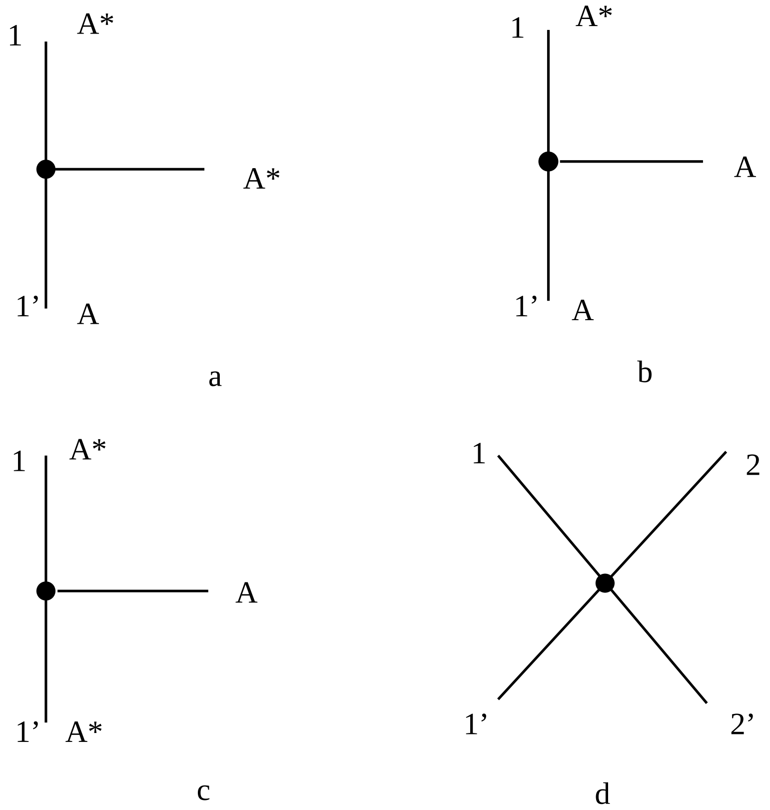

Figure 3: Triple and quartic vertex graphs.

Fig. 3a : Fig. 3b:

$$gT^c_{a_1a_1'}\frac{\beta_1'^2}{\beta_1-\beta_1'}\frac{\kappa_1^*}{\beta_1} \qquad gT^c_{a_1a_1'}\frac{\beta_1^2}{\beta_1-\beta_1'}\frac{\kappa_1}{\beta_1} \tag{3.1}$$

The quartic vertex in (2.3) contributes to Fig. 3d as

$$-g^2T^c_{a_1a_1'}T^c_{a_2a_2'}\frac{\beta_1\beta_2'-\beta_2\beta_1'}{(\beta_1-\beta_1')^2} \tag{3.2}$$

We consider the contribution of these graphs to the two-parton interaction Fig. 4a. There are three contributions as shown in Fig. 4b plus the crossing contributions obtained by interchanging $1'$ and $2'$. The first two terms differ by helicity of the gluon exchanged in s-channel.

The expression for the graph Fig. 4a is ($k = \alpha q' + \beta p + \kappa$)

$$T^c_{a_1 a_1'} T^c_{a_2 a_2'} \frac{g^2}{(2\pi)^4} \int \frac{d^4 k_1}{(k_1^2 + i\epsilon)(k_2^2 + i\epsilon)} \tag{3.3}$$
$$\left(\frac{\beta_1'^2 \beta_2^2 + \beta_1^2 \beta_2'^2}{(\beta_1 - \beta_1')^2 \beta_1 \beta_2} \; \frac{|\kappa_1|^2}{(k_1 - k_1')^2 + i\epsilon} - \frac{(\beta_1 \beta_2' + \beta_2 \beta_1')}{(\beta_1 - \beta_1')^2} \right) \; A^{a_1 a_2}(q_1, q_2, k_1, k_2)$$

$A^{a_1 a_2}$ is the large-scale amplitude factor redefined to be dimensionless by extracting powers of Q.

$$A^{a_1 a_2}(q_1, q_2, k_1, k_2) = \int_{-i\infty}^{+i\infty} \frac{d\nu}{2\pi i} A^{a_1 a_2}(\nu, x_1, \beta_1, x_2, \beta_2) \;\; e^{\nu \xi(Q^2, \kappa^2)} \tag{3.4}$$

We parametrize the momenta as $k_i = \alpha_i q' + \beta_i p + \kappa_i, \quad \kappa_1 + \kappa_2 \approx 0$. The momenta of the (-) modes are to be substituted as $k_i' = \beta_i' p$. We change $\alpha_1 \approx -\alpha_2 = \alpha$ by $\bar{\alpha} = \frac{\alpha s}{|\kappa|^2}$ and obtain

$$\int_{m^2}^{Q^2} \frac{g^2(\kappa^2)}{16\pi^2} \frac{d|\kappa|^2}{|\kappa|^2} \int_0^1 d\beta_1 d\beta_2 \delta(\beta_1 + \beta_2 - \beta_1' - \beta_2') T^c_{a_1 a_1'} T^c_{a_2 a_2'}$$
$$\left\{ \frac{\beta_1'^2 \beta_2^2 + \beta_1^2 \beta_2'^2}{(\beta_1 - \beta_1')^2 \beta_1 \beta_2} J_{111}(\beta_1, -\beta_2, \beta_{11'}) - \frac{\beta_1 \beta_2' + \beta_2 \beta_1'}{(\beta_1 - \beta_1')^2} J_{11}(\beta_1, -\beta_2) \right\}$$
$$A^{a_1 a_2}(\nu, x_1, \beta_1, x_2, \beta_2) \;\; e^{\nu \xi(Q^2, \kappa^2)}. \tag{3.5}$$

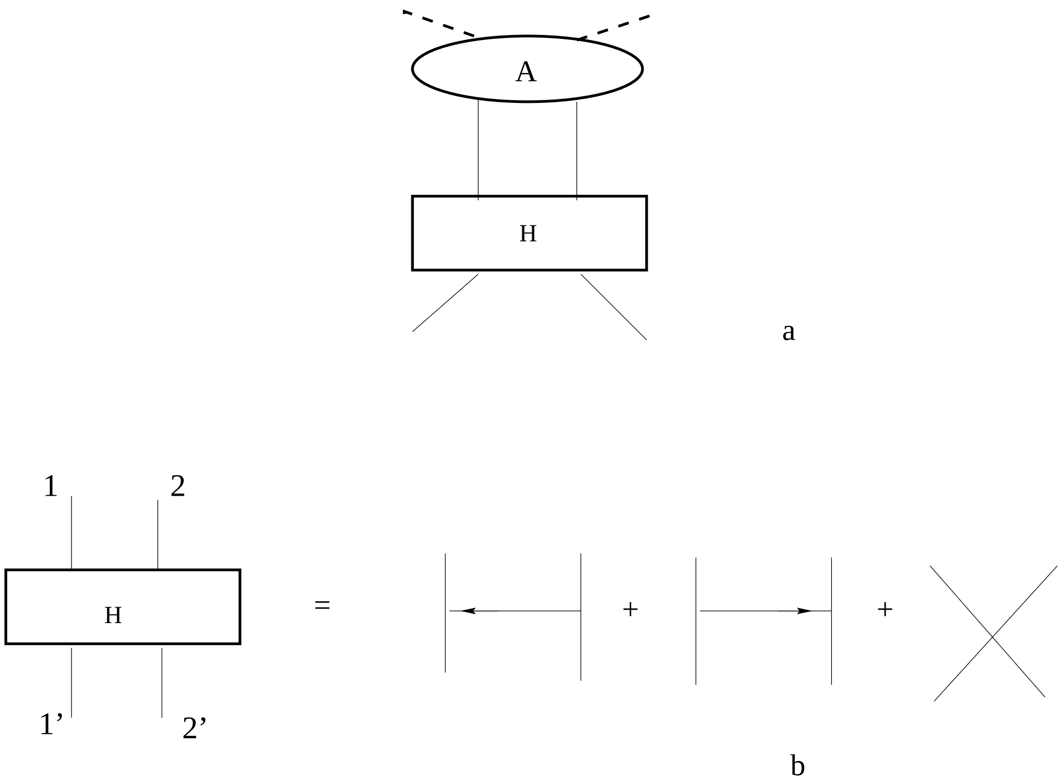

Figure 4: Effective parton interaction.

The J-functions represent standard $\bar{\alpha}$ integrals as explained in detail in Appendix A. The integral over κ leads to the factor $\frac{1}{\nu}$ and we identify the bare interaction kernel $H^{(0)}$ as the expression in the braces. We use the first relation in (8.3) between the J functions to cancel the second order pole at $\beta_1 = \beta_1'$. The remaining first order pole is the infrared singularity being cancelled by the renormalization contribution w_f. As the result we obtain the kernel

$$H^{a_1 a_2}_{a_1' a_2'}(\beta_1, \beta_2; \beta_1', \beta_2') = -T^c_{a_1 a_1'} T^c_{a_2 a_2'} \tag{3.6}$$

$$\left\{ \left[\frac{\beta_1^2}{\beta_1' [\beta_{11'}]_+} J_{11}(\beta_1, \beta_{11'}) \right] + \left[\frac{\beta_2^2}{\beta_2' [\beta_{22'}]_+} J_{11}(\beta_2, \beta_{22'}) \right] + 2 w_g^{(0)} \delta(\beta_1 - \beta_1') \right\} \frac{\beta_1' \beta_2'}{\beta_1 \beta_2}.$$

Besides of illustrating the general scheme the example shows the great simplification of the calculation relying on the effective action. In going to higher loops the main point of effort will lie in the improvement of the effective action, again similar to the Regge case [22]. Still there remains quite a number of two parton channels besides of the one considered here. The calculation can be optimized further and reduced to just three cases by supersymmetry.

3.2 Space-time representation

We outline how to treat the same example in the space-time representation following now the scheme of section 2.3. The vertex in the external field effective action $A^{*(+)}(x_1)A^{*(+)}(x_2)A^{(-)}(x_{1'})A^{(-)}(x_{2'})$ gets a contribution from the original quartic vertex (2.3) and one from contracting two triple vertices by integrating over the quantum fluctuation A_q. The triple vertices in the Bjorken limit efective action (2.3,2.7) corresponding to Fig. 3a, b are

$$-ig \frac{\partial_{1'}^2 \partial_1^{*\perp}}{(\partial_1 \partial_{1'})\partial_1} A_q^{a*} \; (A_1^* T^a A_{1'})|_{x_1 = x_{1'} = x_q}; \quad ig \frac{\partial_1^2 \partial_1^\perp}{(\partial_1 \partial_{1'})\partial_1} A_q^a \; (A_1^* T^a A_{1'})|_{x_1 = x_{1'} = x_q} \tag{3.7}$$

Writing down the extenal field vertex as in (2.20), substituting the asymptotic form of the external fields and doing the interations besides of the ones in the light-ray coordinates we obtain as the particular case of (2.21)

$$\int dz_1 dz_2 dz_{1'} dz_{2'} \left[\frac{\partial_1^2 \partial_{2'}^2 + \partial_{1'}^2 \partial_2^2}{(\partial_1 + \partial_{1'})^2} \tilde{J}_{111} + \frac{\partial_1 \partial_{2'} + \partial_{1'} \partial_2}{(\partial_1 + \partial_{1'})^2} \partial_1 \partial_2 \tilde{J}_0 \right] (A_1^* T^a A_{1'}) \; (A_2^* T^a A_{2'}) \tag{3.8}$$

The first term in the brackets is obtained by contracting the triple vertices by substituting the points $1, 1'$ in the second and $2, 2'$ in the first vertex in (3.7) and vice versa. The second terms emerges from the quartic vertex (2.3) for the case that A_1^*, A_2^* are of $A^{(+)}$ type and the remaining two field of the $A^{(-)}$ type.

The second relation in (8.3) Fourier transformed to light ray variables implies

$$\frac{\partial_1^2 \partial_{2'}^2}{(\partial_1 + \partial_{1'})^2} \tilde{J}_{111} \; + \; \frac{\partial_1^2 \partial_2 \partial_{2'}^2}{(\partial_1 + \partial_{1'})^2} \tilde{J}_0 \; = \; \partial_{1'} \partial_{2'} \; \tilde{J}_{11'}^{(g)}, \tag{3.9}$$

where

$$\partial_1 \partial_2 \tilde{J}_{11'}^{(g)} = -\int_0^1 d\alpha \frac{(1-\alpha)^2}{\alpha} \; \delta(z_{11'} - \alpha z_{12}) \delta(z_{22'}) \tag{3.10}$$

We use this relation and the one obtained by interchanging $1, 1' \leftrightarrow 2, 2'$. The remaining divergence signalled by $\frac{1}{\alpha}$ in the latter integral is cancelled after including the disconnected contribution of order g^2 to the self energy in the two propagators connecting A_1^* with $A_{1'}$ and A_2^* with $A_{2'}$ separately,

$$2 \int dz_1 dz_2 dz_{1'} dz_{2'} \; w_g \; \delta(z_{11'}) \delta(z_{22'}) \; (A_1^* T^a A_{1'}) \; (A_2^* T^a A_{2'}) \tag{3.11}$$

We have applied (2.12) to obtain the above form seemingly connected by colour interaction. w_g is given by (2.10). Adding (3.11) to (3.8) the singular part of w_g results in replacing in $\tilde{J}_{11'}^{(g)}$ (3.10) $\frac{1}{\alpha}$ by $\frac{1}{[\alpha]_+}$, adopting the conventional "plus prescription". We obtain

$$\int dz_1 dz_2 dz_{1'} dz_{2'} [\tilde{J}_{11'}^{(g)} + w_g^{(0)} \tilde{\delta}^{(2)} + (1, 1' \leftrightarrow 2, 2')]$$
$$(A_1^* T^a \partial A_{1'})\ (A_2^* T^a \partial A_{2'}), \quad \text{where } \partial_1 \partial_2 \tilde{\delta}^{(2)} = \delta(z_{11'})\ \delta(z_{22'}) \tag{3.12}$$

We have calculated in the space-time approach the vertex of parallel helicity gluon interaction to order g^2 in the logarithmic approximation. It is given by (3.12) up to the coefficient $\frac{g^2}{8\pi^2} \ln(\frac{\delta^2}{\Delta^2})$. As noticed above by renormalization group the result can be extended to all orders in the leading logarithmic approximation.This leads to the Hamiltonian operator obtained from (3.12) by converting the external fields to operators, creation operators for the fields of type $A^{(+)}$ at points $1, 2$ and annihilation operators for the ones of type $A^{(-)}$ at the points $1', 2'$.

4 Double-logartithmic gluon interaction

4.1 Energy-momentum representation

The scheme of factorizing the asymptotic amplitude into parts determined by large and small scales and of extracting the logarithmic contribution from the sum over intermediate states will be illustrated now in a simpler situation. The combined asymptotics of large Q^2 and $s \gg Q^2$ is of interest in deep-inelastic scattering at small $x \sim \frac{Q^2}{s}$. We avoid here the cases, where double logarithms appear in the Regge asymptotics [23]. We consider the multiple gluon exchange contributing to the amplitude (e.g. of virtual Compton scattering at small $x_{1/2}$) behaving like s^1 on the tree level. In this case the double logarithmic asymptotics can be approached equally well from both the Regge or the Bjorken regions.

Now the loop integrations over the intermediate states in the factorized amplitude Fig. 1,2 have a leading contribution logarithmic both in longitudinal and in transverse momenta. Extracting this leading contribution (and attributing the remainder to non-leading exchange contributions) we obtain that the integrations in all dimensions are standard ones. A double Laplace transform in $\ln \frac{s}{Q^2}$ (conjugated variable $j = 1 + \omega$) and in $\xi(Q^2, m^2)$ (2.8) (conjugated variable ν) is appropriate now.

The dependence on the large scales is only via the product $r^2 = \xi \cdot \ln \frac{s}{Q^2}$ and in the transformed representation we have the dependence only on the product $\rho^2 = \nu\omega$. Actually these products are squares of two-dimensional Lorentz vectors, in particular $\rho^2 = \nu\omega, \underline{\rho} = (\frac{\omega+\nu}{2}, \frac{\omega-\nu}{2})$. These vectors squared are the evolution variable r^2 and its conjugate ρ^2. The evolution proceeds independent of the "rapidity" $\sim \ln(\xi / \ln \frac{s}{Q^2})$ inside the "light cone".

The double logarithmic effective action is obtained from the underlying QCD action by substituting in (2.3) the mode separation (2.6) and reducing the vertices by the conditions

$$\partial_\perp A^{(-)} = 0, \quad \partial(\partial^{-1} A^{(+)}) = 0. \tag{4.1}$$

The latter means that a term involving $A^{(+)}$ is to be neglected unless it is enhanced by its small longitudinal momentum in the denominator. Here also medium modes $A^{(0)}$ have to be separated, carrying transverse momenta κ as large as the ones in the $(+)$ mode but longitudinal momentum fractions β much larger than the ones in the $(+)$ modes and rather compatible with the ones in the (-) mode.

In momentum representation the obtained effective action implies that the quartic vertex gives no leading contribution and the triple vertex is reduced to

$$g\partial_\perp \partial^{-1} A^{(+)a*} i(A^{(0)*} T^a \overleftrightarrow{\partial} A^{(-)}) + \text{c.c.} \tag{4.2}$$

Therefore in the double logarithmic limit the triple vertex graph Fig. 5a vanishes and Fig. 5b and Fig. 5c result in the same expression

$$g f^{a_1 a_{1'} c} \, \frac{\beta_1'}{\beta_1} \;\; \kappa_1^* . \tag{4.3}$$

The only non-vanishing pair interactions arise for gluons with opposite helicities, one interaction producing helicity flip and one without flip in going from $1', 2'$ to $1, 2$. Since the vertices coincide both cases lead to the same expression (compare Fig. 4a)

$$f^{a_1 a_1' c} \; f^{c a_2 a_2'} \frac{g^2}{(2\pi)^4} \int \frac{d^4 k_1}{k_1^2 k_2^2 (k_1 - k_1')^2} \frac{|\kappa_1|^2 \beta_1'^2}{\beta_1^2} \;\; A^{a_1 a_2} \left(\xi(Q^2, \kappa_1^2) \ln \frac{\beta_1 s}{m^2} \right) \tag{4.4}$$

The calculation proceeds as in Sect. 3.1 for the first term in (3.5) involving $|\kappa_1|^2$. In the double logarithmic limit $J_{111}(\beta_1, -\beta_2, \beta_{11'})$ reduces to $\frac{\beta_1}{\beta_1'^2}$ and we obtain in the transformed representation ($\rho^2 = \omega \nu$),

$$f^{a_1 a_1' c} \; f^{c a_2 a_2'} \frac{1}{\rho^2} A^{a_1 a_2}(\rho^2). \tag{4.5}$$

The effective interaction depends on helicity and colour states only; it does not act on longitudinal or transverse momenta.

For a closed formulation of the helicity dependence we introduce the basis vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ for the helicities carried by $A^{(+)*}, A^{(-)}$ and $A^{(-)*}, A^{(+)}$, respectively, and express the interaction operators in terms of Pauli matrices. The spin-flip interaction $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is described by the permutation operator $\mathcal{P} = \frac{1}{2}(I \otimes I + \sigma_i^{(1)} \otimes \sigma_i^{(2)})$ and the non-flip interaction $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ by $-\sigma_3^{(1)} \otimes \sigma_3^{(2)}$. The latter is chosen such that the sum of both operators describes also correctly the vanishing of the interaction between parallel helicities $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. The leading double logarithmic gluon pair Hamiltonian is

$$f^{a_1 a_1 \prime c} \; f^{c a_2 a_2'} \;\; \frac{1}{2} \left(I \otimes I + \sigma_1^{(1)} \otimes \sigma_1^{(2)} + \sigma_2^{(1)} \otimes \sigma_2^{(2)} - \sigma_3^{(1)} \otimes \sigma_3^{(2)} \right) \tag{4.6}$$

We have reproduced a well known result to provide another example for the effective action scheme. The formulation (4.6) may be useful for treating the double-logarithmic multi-gluon exchange by symmetry methods of integrable systems.

4.2 Space-time representation

In the double logarithmic aymptotics the external fields should be specified in the following way: The fields of type $A^{(+)}$ (interacting with the upper blob A in Fig. 1c) are concentrated in the vicinity of the light ray and are almost constant in the light-ray direction (Q is small compared to s, $\delta_s \sim Q^{-1}$),

$$\partial^{-1} A^{(+)} = (A^{(+)} + \mathcal{O}(Q))(\delta^{(2)}(x_\perp)\delta(x_+) + \mathcal{O}(\Delta)) \tag{4.7}$$

The fields of type $A^{(-)}$ (interacting with blob B in Fig. 1c) are almost constant in the directions pointing away off the light ray, however the dependence on the light ray variable z' is a narrow distribution of width $\Delta_s \sim s^{-1}$, approaching a delta function,

$$\partial A^{(-)}(x') \; = \; (A^{(-)}\delta(z'-z_0) + \mathcal{O}(\Delta_s)) \; (\mathrm{const} + \mathcal{O}(m)) \tag{4.8}$$

$A^{(\pm)}$ on the right-hand sides of (4.7,4.8) are constants; they should not be identified with similar symbols in other sections.

Consider now the vertex $A^{(+)}(x_1)A^{(+)}(x_2)A^{(-)}(x_{1'})A^{(-)}(x_{2'})$. The integrations over the coordinates pointing away off the light ray are done in logarithmic approximation as above. Unlike the Bjorken asymptotics now also the dependence on the light ray coordinates is specified in the asymptotic external fields and therefore also the integration over these variables can be done. We shall do these integrations approximately extracting the second logarithm of the large scale ratio. We rely on the results of the Bjorken asymptotics for the gluon interaction of parallel helicity (3.8) and of antiparallel helicity given in Appendix B (9.1). Now we use the information about the z dependence specified in (4.7,4.8) and notice that the derivatives ∂_1, ∂_2 act on (almost) constant fields. We conclude immediately that the considered external field vertex vanishes in the case of parallel helicities (3.8) and in the antiparallel helicity case (9.1) it reduces to

$$\begin{gathered}\int dz_1 dz_2 dz_{1'} dz_{2'} \delta(z_{1'}-z_0)\delta(z_{2'}-z_0)\partial_1\partial_2 \tilde{J}_{111} \\ [(A^{(+)*}T^a A^{(-)})(A^{(+)}T^a A^{(-)*}) + (A^{(+)*}T^a A^{(-)*})(A^{(+)}T^a A^{(-)})]\end{gathered} \tag{4.9}$$

We extract the logarithmic contribution in the integral over $\tilde{J}_{111}$ for $z_{1'}, z_{2'} \to z_0$ appearing as a divergence $\sim \alpha_3^{-1}$ in the α representation (8.3, 5.4).

Up to the coefficient $\sim g^2(\ln(\delta/\Delta))\ln(\delta_s/\Delta_s)$ the resulting vertex is given by the bracket in (4.9) involving the double-logarithmic "fields", i.e. the constats $A^{(\pm)}$ carrying merely the colour and helicity degrees of freedom. In order to extend the result to all orders in the double-logarithmic approximation we convert the obtained vertex into an Hamiltonian operator by defining the "fields" $A^{(\pm)}$ to be creation and annihilation operators. The resulting operator is of course equivalent to the result obtained in the momentum calculation (4.6). From the latter being a matrix representation we recover the operator representation (4.9) by multiplication by the helicity state vectors composed of the creation and annihilation operators, $(A^{*(+)}, A^{(+)})_1 \otimes (A^{*(+)}, A^{(+)})_2$ from the left and $(A^{(-)}, A^{*(-)})_1^T \otimes (A^{(-)}, A^{*(-)})_2^T$ from the right.

5 Interaction operators in the Bjorken limit

The momentum kernels or the external field vertices calculated above in the tree approximation are extended to all orders in the leading logarithmic approximation. This extension is represented in terms of the evolution in "euclidean time" $\xi(Q^2, m^2)$, describing the approach to the Bjorken asymptotics, which can be formulated in Hamiltonian operator language. Let the parton fields $A^{(-)}, f^{(-)}, \tilde{f}^{(-)}$ act as annihilation operators and $A^{(+)}, f^{(+)}, \tilde{f}^{(+)}$ as creation operators with the contractions (longitudinal momentum representation)

$$< 0|A^{a(-)}(\beta)A^{b(+)*}(\beta')|0 >= \frac{1}{2}\delta(\beta-\beta')\delta^{ab},$$

$$< 0|A^{a(-)}(\beta)A^{b(+)}(\beta')|0 >= 0,$$
$$< 0|f^{\alpha(-)}(\beta)f^{\beta(+)*}(\beta')|0 >= \beta\delta(\beta-\beta')\delta^{\alpha\beta}. \tag{5.1}$$

This corresponds to the following kinetic terms (light-ray position representation)

$$\int dz\{2(A^{a*(+)}A^{a(-)}) - if^{\alpha(+)*}\partial^{-1}f^{\alpha(-)} + \text{c.c.} + ...\} \tag{5.2}$$

The periods stand for the terms of other fermion chirality and flavours. c.c. does not exchange (+) and (-) and acts as conjugation otherwise (normal ordering). The parton interaction Hamiltonian operators can be obtained from the momentum kernels and also from the external field vertices,

$$\begin{aligned}\int d^4\beta\delta(\beta_1+\beta_2-\beta_{1'}-\beta')(-\beta_1\beta_2)^{-1})(A^{(+)*}(\beta_1)T^aA^{(-)}(\beta_{1'}))\\ (A^{(+)*}(\beta_2)T^aA^{(-)}(\beta_{2'}))\ \ H(\beta_1,\beta_2;\beta_{1'},\beta_{2'}) =\\ \int d^4z(A^{(+)*}(z_1)T^aA^{(-)}(z_{1'}))\ (A^{(+)*}(z_2)T^aA^{(-)}(z_{2'}))\ \tilde{H}(z_1,z_2;z_{1'},z_{2'}).\end{aligned} \tag{5.3}$$

There is a convenient representation of the kernels in terms of Feynman parameter integrals. As shown in Appendix A, the standard $\bar{\alpha}$ integrals $J(\beta,...)$ can be written in Feynman parameter form allowing to do the Fourier transformation rather easily.

We list the operators of the leading parton interaction in the QCD Bjorken limit, the momentum kernels of which are well known, in the light-ray and Feynman parameter form. We restrict ourselves to the ones involving gluons (A) and/or only one flavour and one chirality of quarks (f). We use the abbreviations (compare Appendix A)

$$\begin{aligned}\partial_1\partial_2\tilde{J}^{(p)}_{11'} &= \int_0^1\frac{d\alpha}{[\alpha]_+}\ \chi_1^{(p)}(\alpha)\ \delta(z_{11'}-\alpha z_{12})\delta(z_{22'}),\\ \chi_1^{(0)} &= \alpha(1-\alpha), \chi_1^{(f)} = -(1-\alpha), \chi_1^{(g)} = -(1-\alpha)^2,\\ \partial_1\partial_2\tilde{J}_{n_1n_2n_3} &= \frac{\Gamma(n_1+n_2+n_3-1)}{\Gamma(n_1)\Gamma(n_2)\Gamma(n_3)}\int_0^1 d\alpha_1d\alpha_2d\alpha_3\delta(\sum\alpha_i)\\ &\alpha_1^{n_1}\alpha_2^{n_2}\alpha_3^{n_3}\ \delta(z_{11'}-\alpha_1z_{12})\delta(z_{22'}+\alpha_2z_{12}),\\ \partial_1\partial_2\tilde{J}_0^{(g)} &= \int_0^1 d\alpha\ \chi_0^{(p)}(\alpha)\ \delta(z_{11'}-\alpha z_{12})\delta(z_{22'}+(1-\alpha)z_{12}),\\ \chi_0^{(0)} &= 1,\ \chi_0^{(g)} = \alpha(1-\alpha),\qquad \partial_1\partial_2\delta^{(2)} = \delta(z_{11'})\delta(z_{22'}).\end{aligned} \tag{5.4}$$

The case of parallel helicity gluon interaction has been done as the example in section 3. In the cases of antiparallel helicity interactions some further transformations of J kernels (8.3) are applied for presenting the result in a convenient form. The case of gluons of antiparallel helicities is treated in Appendix B.

The position arguments of the fields will be abbreviated as indices 1, 1′, 2, 2′. We suppress the label ($\pm$) distinguishing creation and annihilation operators, since the fields at points $1, 2$ act always as creation and the fields at the points $1', 2'$ always as annihilation operators. The integration over the positions, summation over colour indices and operator normal ordering is implied.

parallel helicity interactions

$$\begin{aligned}\{4[\tilde{J}^{(g)}_{11'} + w_g^{(0)}\delta^{(2)}](A_1^*T^a\partial A_{1'}) - 2i[\tilde{J}^{(f)}_{11'} + w_f^{(0)}\delta^{(2)}](f_1^*t^af_{1'})\}\\ [-2(A_2^*T^a\partial A_{2'}) + i(f_2^*t^af_{2'})]\\ -4i\tilde{J}^{(0)}_{11'}(f_1^*t^\alpha A_{1'})\ (A_2^*t^{*\alpha}f_{2'}) + \text{c.c.}\end{aligned} \tag{5.5}$$

anti-parallel gluon interactions

$$\begin{aligned}\{8[\tilde{J}^{(g)}_{11'} + w^{(0)}_g \delta^{(2)}] \ + 4\tilde{J}_{221} - 8\tilde{J}_{112}\} \\ (A^*_1 T^a \partial A_{1'})(A_2 T^a \partial A^*_{2'}) \\ +4\tilde{J}_{221}(A^*_1 T^a \partial A^*_{1'})(A_2 T^a \partial A_{2'}) + \text{c.c.}\end{aligned} \tag{5.6}$$

anti-parallel helicity quark interactions (one chirality, one flavour)

$$\begin{aligned}\{-2[\tilde{J}^{(f)}_{11'} + w^{(0)}_f \delta^{(2)}] + \tilde{J}_{111}\}(f^*_1 t^a f_{1'})(f_2 t^a f^*_{2'}) \\ +2\tilde{J}^{(g)}_0 (f^*_1 t^a f_2)(f_{1'} t^a f^*_{2'}) + \text{c.c.}\end{aligned} \tag{5.7}$$

anti parallel helicity quark - gluon interactions

$$\begin{aligned}-4i[\tilde{J}^{(g)}_{11'} + w^{(0)}_g \delta^{(2)}](A^*_1 T^a \partial A_{1'}) \ (f_2 t^a f^*_{2'}) \\ -4i[\tilde{J}^{(f)}_{11'} + w^{(0)}_f \delta^{(2)}](f^*_1 t^a f_{1'})(A_2 T^a \partial A^*_{2'}) \\ +4i\tilde{J}_{211}[(f^*_1 t^\alpha \partial A^*_{1'}) \ (A_2 t^{*\alpha} f_{2'}) - (A^*_1 T^a \partial A_{1'})(f_2 t^a f^*_{2'})] \\ -8i\tilde{J}_{111}(A^*_1 T^a \partial A_{1'})(f_2 t^a f^*_{2'}) \quad - 4i\tilde{J}^{(g)}_0 \ (f^*_1 t^\alpha A_2)\partial(f_{1'} t^{*\alpha} A^*_{2'}) + \text{c.c.}\end{aligned} \tag{5.8}$$

annihilation-type interactions

$$\begin{aligned}2i\tilde{J}^{(0)}_{11'}[(A^*_1 t^{*\alpha} \partial f_{1'})(A_2 t^\alpha f^*_{2'}) + (f^*_1 t^\alpha \partial A_{1'}) \ (f_2 t^{*\alpha} A_{2'})] \\ +2i\tilde{J}^{(g)}_0[(A^*_1 T^a \overleftrightarrow{\partial} A_2)(f_{1'} t^a f^*_{2'}) + (f^*_1 t^a f_2) \ (A_{1'} T^a \overleftrightarrow{\partial} A^*_{2'}] \\ -2i\tilde{J}_{111}[(f^*_1 t^\alpha \partial A^*_{1'})(f_2 t^{*\alpha} A_{2'}) + (A^*_1 t^\alpha f^*_{1'})(A_2 t^{*\alpha} \partial f_{2'})] \\ -2i\tilde{J}_{211}[(f^*_1 t^\alpha \partial A^*_{1'})(f_2 t^{*\alpha} A_{2'}) + (A^*_1 t^\alpha \partial f^*_{1'})(A_2 t^{*\alpha} f_{2'}) \\ -(\partial f^*_1 t^\alpha \partial A_{1'})(f_2 t^\alpha A^*_{2'})] + \text{c.c.}\end{aligned} \tag{5.9}$$

6 Symmetries of the parton interactions

6.1 Conformal symmetry

It is well known that the leading order parton interactions are conformally symmetric. In the present light-ray formulation this symmetry is manifest. The interaction operators (5.5 - 5.9) as well as the kinetic operators (5.2) are invariant under the transformations acting on the light-ray positions as

$$z \to \tilde{z} = \frac{az + b}{cz + d} \tag{6.1}$$

(a, b, c, d real) and on the fields (conformal primaries of weights $\frac{1}{2}$ and 1)

$$A \to \frac{1}{cz + d} A(\tilde{z}), \quad f \to \frac{1}{(cz + d)^2} f(\tilde{z}). \tag{6.2}$$

Since translation and scale symmetries are obvious only the infinitesimal proper-conformal transformations ($a = d = 1, b = 0, c$ infinitesimal) are to be checked. The action on the operator terms for given parton types and helicities is irreducible, i.e. all terms are involved in the mutual cancellation of symmetry variations.

Let us do the example of quarks of parallel helicity. The essential term can be written (in simplified notation disregarding colour) as

$$\int d^4 z \int_0^1 d\alpha \frac{1-\alpha}{\alpha} \delta(z_{11'} - \alpha z_{12}) \delta(z_{22'}) \partial^{-1} f^{(+)*}(z_1) \partial^{-1} f^{(+)*}(z_2)$$

$$f^{(-)}(z_{1'})f^{(-)}(z_{2'}) \tag{6.3}$$

The transformation acts on the fields and results in

$$\int d^4z \int_0^1 d\alpha \frac{1-\alpha}{\alpha}\delta(z_{11'}-\alpha z_{12})\delta(z_{22'})\partial^{-1}\left(\frac{1}{(1+cz_1)^2}f^{(+)*}(\tilde{z}_1)\right)$$
$$\partial^{-1}\left(\frac{1}{(1+cz_2)^2}f^{(+)*}(\tilde{z}_2)\right)\frac{1}{(1+cz_{1'})^2}f^{(-)}(\tilde{z}_{1'})\frac{1}{(1+cz_{2'})^2}f^{(-)}(\tilde{z}_{2'}) \tag{6.4}$$

We have to check that the latter two expressions coincide up to $\mathcal{O}(c^2)$. We have $\partial_{\tilde{z}} = (1+cz)^2\partial_z$, $d\tilde{z} = (1+cz)^{-2}dz$ and

$$\int_0^1 d\alpha \frac{1-\alpha}{\alpha}\delta(z_{11'}-\alpha z_{12})\delta(z_{22'}) = (1+cz_1)^{-2}(1+cz_2)^{-2}$$
$$\cdot \int_0^1 d\alpha \frac{1-\alpha}{\alpha}\delta(\tilde{z}_{11'}-\alpha\tilde{z}_{12})\delta(\tilde{z}_{22'}) \tag{6.5}$$

By substituting these relations we see that (6.4) transforms into (6.3) with the variables z changed to $\tilde{z}$.

The check is equally simple for the other terms involving only fermions in the initial state (i.e. as (-) modes). Supersymmetry allows to avoid the check in the less simple cases. Indeed, in the case of only one fermion flavour and chirality in the adjoint gauge group representation supersymmetry holds and connects the mentioned simple terms with all the remaining ones. In this case the proof of conformal symmetry is extended from the fermionic to all terms by supersymmetry. Now this implies also conformal symmetry of the corresponding operators in the non-supersymmetric case, because the difference is only in gauge group factors passive in conformal transformations.

Investigating the conformal symmetry one can rely on the observation that a kernel, being a conformal symmetric 4-point-function of conformal primaries with weights $\ell_1, \ell_2, \ell_{1'}, \ell_{2'}$ in the corresponding points with the restriction $\ell_1+\ell_2+\ell_{1'}+\ell_{2'} = 2$ and with the integration range for $z_2 > z_1 : z_2 > z_{1'} > z_{2'} > z_1$ and for $z_1 > z_2 : z_1 > z_{2'} > z_{1'} > z_2$, has the general form

$$\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta(\sum \alpha_i - 1)\ \alpha_1^{a_{21'}}\ \alpha_2^{a_{12'}}\ (1-\alpha_1)^{a_{11'}}\ (1-\alpha_2)^{a_{22'}}\ \alpha_3^{a_{1'2'}}$$
$$\varphi(\frac{\alpha_3}{\alpha_1\alpha_2})\ \ \delta(z_{11'}-\alpha_1 z_{12})\ \ \delta(z_{22'}+\alpha_2 z_{12}) \tag{6.6}$$

where the exponents obey

$$\sum_{i, i\neq j} a_{ij} = -2\ell_j, \qquad a_{ij} = a_{ji} \quad i,j = 1,2,1',2'. \tag{6.7}$$

and φ is an arbitrary function.

We notice that the inverse power z_{ij}^{-n} and the $(n-1)$st derivative of $\delta(z_{ij})$ have the same conformal transformation property. Therefore also distributions can be substituted for φ obtained from the above function by replacing in its power expansion inverse powers of α_i by the corresponding derivative of $\delta(\alpha_i)$.

The action of this kernel on functions of $z_{1'}, z_{2'}$ is by integrating its product with the kernel with repect to both primed arguments over the full light ray, i.e. the real axis. The above form of the kernel is to be compared with (5.4).

6.2 Helicity symmetry

The operators (5.5 - 5.9) are invariant also under the $SU(1,1)$ transformation acting on helicities, on fields of type $A^{(+)}$, the position argument of which we agreed to denote by 1 or 2,

$$\begin{pmatrix} A_1^* \\ A_1 \end{pmatrix} \to \begin{pmatrix} \tilde{A}_1^* \\ \tilde{A}_1 \end{pmatrix} = \hat{V}(a_1, a_2, a_3) \begin{pmatrix} A^* \\ A \end{pmatrix}. \tag{6.8}$$

where

$$\hat{V}(a_1, a_2, a_3) \;=\; \exp[\frac{1}{2}(a_1\sigma_1 + a_2\sigma_2 - ia_3\sigma_3] \tag{6.9}$$

The fields of type $A^{(-)}$ transform analogously with $\hat{V}$ replaced by $\hat{V}^{+-1}$. In particular the light ray kinetic terms (5.2) are left invariant.

The products of a field of type $A^{(+)}$ and one of type $A^{(-)}$ (say at positions 1 and $1'$) decompose into the scalar and vector components, where the (2+1) vectors of this group can be constructed in two ways, using $(\sigma_r) = (\sigma_1, \sigma_2; \sigma_3)$ or using $(\tilde{\sigma}_r) = (\sigma_1, \sigma_2; -\sigma_3)$. The two types of vectors v and $\tilde{v}$ are related to each other by a parity transformation, analogous to time reversal. In particular the vectors constructed with $\frac{1}{2}(\sigma_r \pm \tilde{\sigma}_r)$ are even or odd with repect to that parity. The subspaces of even or odd vectors are two- or one-dimensional.

Thus the four states in the tensor product $\begin{pmatrix} A_1^* \\ A_1 \end{pmatrix} \otimes \begin{pmatrix} A_{1'}^* \\ A_{1'} \end{pmatrix}$ decompose according to

$$R = (A_1^* \;\; A_1)\hat{R}\begin{pmatrix} A_{1'}^* \\ A_{1'} \end{pmatrix}, R = \mathcal{V}, \mathcal{A}, \mathcal{T}^{\pm} \tag{6.10}$$

into the scalar representation,

$$\hat{R} = \hat{\mathcal{V}} = \sigma_1, \quad R = \mathcal{V} = A_1^* A_{1'} + A_1 A_{1'}^*, \tag{6.11}$$

the one-dimensional odd-vector representation,

$$\hat{R} = \hat{\mathcal{A}} = \sigma_1\sigma_3, \quad R = \mathcal{A} = A_1^* A_{1'} - A_1 A_{1'}^*, \tag{6.12}$$

and the two-dimensional even-vector representation,

$$\hat{R} = \hat{\mathcal{T}}^{\pm} = \sigma_1\sigma_{\pm}, \;\; \mathcal{T}^+ = A_1^* A_{1'}^*, \;\; \mathcal{T}^- = A_1 A_{1'}. \tag{6.13}$$

If the indices 1 and $1'$ refer to out (+) and in (-) going partons, then the product representations describe transitions, where one has

$\mathcal{V}_s$: no helicity flip, helicity independent ($\delta_{\lambda\lambda'}$),

$\mathcal{A}_s$: no helicity flip, helicity dependent, ($\lambda\delta_{\lambda\lambda'}$),

$\mathcal{T}_s^{\pm}$: helicity flip from $\pm$ to $\mp$.

If the position indices would be instead 1 and 2 referring both to outgoing states, then the product representations would describe states, where one would have

$\mathcal{V}_t$: opposite helicities, symmetric,

$\mathcal{A}_t$: opposite helicities, antisymmetric,

$\mathcal{T}_t^{\pm}$: parallel helicities of two orientations.

The invariant out of two vectors (of any types) is built as

$$g^{r,s} v_r v_s, \quad r, s = 1, 2, 3, (g^{r,s}) = \mathrm{diag}(1, 1, -1) \tag{6.14}$$

In this way three different symmetric four-parton helicity operators

$$\begin{pmatrix} A_1^* \\ A_1 \end{pmatrix}^T \begin{pmatrix} A_2^* \\ A_2 \end{pmatrix}^T \hat{O}_{11',22'} \begin{pmatrix} A_{1'}^* \\ A_{1'} \end{pmatrix} \begin{pmatrix} A_{2'}^* \\ A_{2'} \end{pmatrix} \tag{6.15}$$

can be constructed with $\hat{O}_{1,1',22'}$ being replaced by the projectors

$$\hat{\mathcal{V}}_s = \hat{\mathcal{V}}^{11'} \otimes \hat{\mathcal{V}}^{22'}, \hat{\mathcal{A}}_s = \hat{\mathcal{A}}^{11'} \otimes \hat{\mathcal{A}}^{22'}, \hat{\mathcal{T}}_s = \hat{\mathcal{T}}^{+11'} \otimes \hat{\mathcal{T}}^{-22'} + \hat{\mathcal{T}}^{-11'} \otimes \hat{\mathcal{T}}^{+22'}. \tag{6.16}$$

The latter three operators correspond to definite $SU(1,1)$ helicity states in s-channel ($11' \to 22'$). The operators with definite helicity states in other channels can be written in the same way. We quote the crossing relations between the channels s ($11' \to 22'$), u ($12' \to 21'$) and t ($1'2' \to 12$) [24]

$$\begin{aligned} \hat{\mathcal{V}}_s = \hat{\mathcal{V}}_u &= \hat{\mathcal{T}}_t + \hat{\mathcal{V}}_t + \hat{\mathcal{A}}_t, \\ \hat{\mathcal{A}}_s = -\hat{\mathcal{A}}_u &= \hat{\mathcal{T}}_t - \hat{\mathcal{V}}_t - \hat{\mathcal{A}}_t, \\ \hat{\mathcal{T}}_s = \hat{\mathcal{T}}_u &= \hat{\mathcal{V}}_t - \hat{\mathcal{A}}_t. \end{aligned} \tag{6.17}$$

Obvously, the parallel helicity operators (5.5) are of the form $\mathcal{T}_t$ and the antiparallel helicity operators (5.6- 5.9) decompose into $\mathcal{V}_t$ and $\mathcal{A}_t$. In particular all pure gluonic operators can be written in the following s-channel form:

$$\begin{gathered} \begin{pmatrix} A_1^* \\ A_1 \end{pmatrix}^T \begin{pmatrix} A_2^* \\ A_2 \end{pmatrix}^T \hat{O}_{11',22'}\, \partial \begin{pmatrix} A_{1'}^* \\ A_{1'} \end{pmatrix} \partial \begin{pmatrix} A_{2'}^* \\ A_{2'} \end{pmatrix}, \\ \hat{G}_{11'22'} = (T^a_{11'} \otimes T^a_{22'}) \\ \left\{ -8[\tilde{J}^{(g)}_{11'} + w_g^{(0)} \delta^{(2)}] \hat{\mathcal{V}}_s + 2[\tilde{J}_{221}](\hat{\mathcal{V}}_s - \hat{\mathcal{A}}_s + 2\hat{\mathcal{T}}_s) - 4[\tilde{J}_{112}](\hat{\mathcal{V}}_s - \hat{\mathcal{A}}_s) \right\} \end{gathered} \tag{6.18}$$

The operator

$$\hat{\mathcal{V}}_s - \hat{\mathcal{A}}_s + 2\hat{\mathcal{T}}_s = \hat{\mathcal{V}}_t = \sigma_1^{(11')} \otimes \sigma_1^{(22')} [I^{(11')} \otimes I^{(22')} + g^{rs} \sigma_r^{(11')} \otimes \sigma_s^{(22')}] \tag{6.19}$$

has appeared already above in the double-logarithmic asymptotics (4.6).

6.3 Two-parton eigenstates

Two-parton states on which the above operators act have the form

$$|\phi_{12}> = \sum_{P_i} \int dz_1 dz_2\ \phi^{P_1 P_2}(z_1, z_2)\ A^{(+)}_{P_1}(z_1)\ A^{(+)}_{P_2}(z_2)\ |0> . \tag{6.20}$$

$P = (p, \lambda)$ labels the parton type p (chirality and flavour) and the helicity λ, i.e. $A_P = (A^*, A, f^*, f, \tilde{f}^* ...)$. The action of the symmetries on the fields induces an action on ϕ. In particular the action of the one-dimensional conformal transformations (6.1, 6.2) on ϕ is generated by $S^a = S_1^a + S_2^a, a = \pm, 0$

$$S_1^- = \partial_1, \quad S_1^+ = z_1^2 \partial_1 + 2 s_{p1} z_1, \quad S_1^0 = z_1 \partial_1 + s_{p1}, \tag{6.21}$$

where $s_A = \frac{1}{2}, s_f = 1$.

Conformal symmetry is used to find the eigenstates and eigenvalues of the above operators. Among the states with the same eigenvalues the ones of lowest weight are distinguished,

$$(S_1^- + S_2^-)\ \phi^{P_1 P_2}(z_1, z_2) = 0,$$

$$
\begin{aligned}
(S_1^0+S_2^0)\ \phi^{P_1P_2}(z_1,z_2) = (n+s_{p1}+s_{p2})\ \phi^{P_1P_2}(z_1,z_2),\\
\phi^{P_1P_2}(z_1,z_2) \ = \ C_n^{P_1P_2}\cdot(z_{12})^n.
\end{aligned}
\tag{6.22}
$$

Due to the helicity symmetry the eigenstates can be chosen to be definite helicity states $\mathcal{T}^{\pm}, \mathcal{V}, \mathcal{A}$, therefore

$$
\begin{aligned}
C_{n,T^{\pm}}^{(p_1,\lambda_1)(p_2,\lambda_2)} &= \tilde{C}_{n,T^{\pm}}^{p_1p_2}\ \frac{1\pm\lambda_1}{2}\delta_{\lambda_1,\lambda_2},\\
C_{n,V}^{(p_1,\lambda_1)(p_2,\lambda_2)} &= \tilde{C}_{n,V}^{p_1p_2}\ \delta_{\lambda_1,-\lambda_2},\\
C_{n,A}^{(p_1,\lambda_1)(p_2,\lambda_2)} &= \tilde{C}_{n,A}^{p_1p_2}\ \lambda_1\delta_{\lambda_1,-\lambda_2}.
\end{aligned}
\tag{6.23}
$$

Clearly, the states $\mathcal{T}^{\pm}$ have the same eigenvalue.

If flavour or chirality do not coincide, $p_1 \neq p_2$, then still the diagonalization in that subspace has to be performed.

Comparing to the Regge limit we see that conformal symmetry holds in the corresponding logarithmic approximation to both limits. In the Regge case the operators depend on the transverse position (impact parameter) and the conformal symmetry acts on the plane by Möbius transformations (based on $sl(2,C)$). The operators of reggeon interactions allow for factorization of the holomorphic and anti-holomorphic dependences on the complex position variable. The leading reggeized gluons do not carry helicity and there is no similarity regarding to the helicity symmetry.

7 Discussion

The eigenvalues are calculated easily in the α representation (5.4) reproducing the well known results for the sets of anomalous dimensions in the exchange channels of parallel helicities $\mathcal{T}_t$ related to transversity asymmetry or chirality odd structure functions like h_1, F_3^{γ}, symmetric antiparallel helicity $\mathcal{V}_t$, related to unpolarized parton distributions, and antisymmetric antiparallel helicity $\mathcal{A}_t$, related to helicity asymmetry and structure functions g_1. The parton states are directly related to the conventional local light-cone operators composed out of the gluon field strength operator and the Dirac field operators with derivatives acting on them. In the leading twist case the field strength enters with one index contracted with q' and one being transverse and expessing the polarization state, the chirality and polarization states of the quarks are selected by the known γ matrix projectors, among them a factor $\gamma_\mu q'^\mu$, and the derivatives are all longitudinal, $q'^\mu\partial_\mu$. The light-ray wave functions are directly related to the derivative structure of the operator: The number of derivatives acting on the ith operator is the power of z_i in the polynomial $\phi(z_i)$.

The eigenstates and the eigenvalues of the two-parton hamiltonians allow to construct the contribution from two parton exchange, i.e. of twist two, to the amplitude in the Bjorken asymptotics. The multi-parton exchange (higher twist) contribution leads to a multi-body quantum problem in one dimension the hamiltonian of which is the sum of the above pair-interaction hamiltonians. The solution of this stationary state quantum problem allows to construct the corresponding higher twist contribution to the Bjorken limit amplitude.

This appears again in parallel to the Regge case. In both cases some of the arising multi-body quantum systems turn out to be completely integrable (without or with simplification of the gauge group factors by taking the $N \to \infty$ limit). In these cases the involved pair hamiltonians are particular operators out of complete sets of integrals of motion which can be constructed from solutions of the Yang-Baxter equation.

The pair interaction hamiltonians of the parallel helicity partons (5.5) (without the gauge group factors) correspond to integrable multi-body systems with nearest neighbour (chain) interactions constructed from $sl(2)$ symmetric solutions of the Yang-Baxter equation. Interesting three-parton channels have been studied in [19].

The parallel helicity operators altogether (disregarding the gauge group factors) appear in the integrable homogeneous closed chain constructed out of the ($\ell = \frac{1}{2}, b = \frac{1}{2}$) representation of the superconformal group $sl(2|1)$ [25].

The natural question arises whether integrable systems can be constructed the hamiltonians of which coincide with the ones of the other helicity configurations $\mathcal{V}_t, \mathcal{A}_t$.

We have treated the high energy Bjorken asymptotics of QCD amplitudes choosing the non-standard viewpoint of the effective action concept. While this does not add new results to what is well known about scale dependence of composite operators or (ordinary and skewed) parton distributions it allows to draw interesting parallels to the Regge asymptotics and provides a compact and uniform treatment of all exchange channels. A motivation is to rely on the observed parallels in improving the understanding of the more involved Regge asymptotics.

The results have been formulated in terms of Hamiltonian operators involving light-ray kernels in a convenient Feynman parameter representation. In this operator form the symmetries of the one-dimensional effective parton interaction are manifest, providing a convenient basis for treating the mutiparton exchange contributions of higher twist by symmetry methods.

The symmetries of the asymptotic effective interaction are derived from the symmetries of the underlying theory. The impact of extended symmetries (compared to QCD) on the asymptotic interactions is a question of interest, since the answer may be helpful in treating multiple exchanges. With this motivation we are going to consider the models of $\mathcal{N} = 1, 2, 4$ supersymmetric Yang-Mills theory in a forthcoming paper. The present effective action formulation allows for a simple and uniform treatment of the Bjorken asymptotics of those models.

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8 Appendix A

The general definition of the $\bar{\alpha}$ integrals appearing in the leading loop contributions are

$$J_{n_1 n_2 \dots n_N}(x_1, x_2, ..., x_N) = \int_{-\infty}^{+\infty} \frac{d\bar{\alpha}}{2\pi i} [\bar{\alpha} x_1 + 1 - i\epsilon]^{-n_1} [\bar{\alpha} x_2 + 1 - i\epsilon]^{-n_2} \\ ...[\bar{\alpha} x_N + 1 - i\epsilon]^{-n_N}. \quad (8.1)$$

n_i are positive integers. The Feynman parameter representation is

$$J_{n_1 n_2 \dots}(x_1, x_2, ...) = \frac{\Gamma(\sum n_i - 1)}{\Gamma(n_1)\Gamma(n_2)...\Gamma(n_N)} \int \mathcal{D}^{(N)}\alpha \; \alpha_1^{n_1 - 1} \alpha_2^{n_2 - 1} \\ ...\alpha_N^{n_N - 1} \; \delta(\sum \alpha_i x_i) \quad (8.2)$$

In the latter integral the varibles α_i range from 0 to 1 and $\mathcal{D}^{(N)}\alpha = d\alpha_1 ... d\alpha_N \delta((1 - \sum \alpha_i)$.

By decomposition into simple fractions one can reduce the number of factors in the denominator of (8.1). In this way one derives relations like

$$J_{12}(x_1, x_2) = \frac{x_1}{x_1 - x_2} J_{11}(x_1, x_2),$$

$$
\begin{aligned}
J_{111}(x_1,x_2,x_3) &= \frac{x_1}{x_{23}}J_{11}(x_1,x_2) - \frac{x_3}{x_{23}}J_{11}(x_1,x_2),\\
J_{112}(x_1,x_2,x_3) &= \frac{-x_1x_2}{x_{13}x_{23}}J_{11}(x_1,x_2) + \left(\frac{x_2}{x_{23}} + \frac{x_1}{x_{13}}\right)J_{111}(x_1,x_2,x_3),\\
J_{221}(x_1,x_2,x_3) &= \frac{x_3^2}{x_{13}x_{23}}J_{111}(x_1,x_2,x_3)\\
&\quad -\frac{x_1x_2}{x_{12}^2}\left(\frac{x_2}{x_{23}} + \frac{x_1}{x_{13}}\right)J_{11}(x_1,x_2).
\end{aligned}
\tag{8.3}
$$

In the above calculations we encounter $\bar{\alpha}$ integrals with three arguments ($x_1 = \beta_1, x_2 = -\beta_2, x_3 = \beta_{11'}$), two cases with two arguments ($x_1 = \beta_1, x_2 = \beta_{11'}$ and $x_1 = \beta_1, x_2 = -\beta_2$) and with one argument, which is just a δ function.

Under Fourier transformation

$$
\mathcal{F}[\Phi(\beta_1,\beta_2,\beta_{1'},\beta_{2'})] = \tilde{\Phi}(z_1,z_2,z_{1'},z_{2'}) \tag{8.4}
$$

defined by

$$
\begin{aligned}
\tilde{\Phi}(z_1,z_2,z_{1'},z_{2'}) &= \int \frac{d\beta_1 d\beta_2 d\beta_{1'} d\beta_{2'}}{(2\pi)^2}\delta(\beta_1+\beta_2-\beta_{1'}-\beta_{2'})\\
&\exp[i(z_{1'}\beta_{1'} + z_{2'}\beta_{2'} + -z_1\beta_1 - z_2\beta_2)]]\Phi(\beta_1,\beta_2,\beta_{1'},\beta_{2'})
\end{aligned}
\tag{8.5}
$$

the expressions involving J appearing in the interaction kernels transform as

$$
\begin{aligned}
\mathcal{F}[J_{n_1n_2n_3}(\beta_1,-\beta_2,\beta_{11'})] &= \frac{\Gamma(n_1+n_2+n_3-1)}{\Gamma(n_1)\Gamma(n_2)\Gamma(n_3)}\int \mathcal{D}^{(3)}\alpha\\
&\alpha_1^{n_1-1}\alpha_2^{n_2-1}\alpha_3^{n_3-1}\ \delta(z_{11'}-\alpha_1 z_{12})\ \delta(z_{22'}+\alpha_2 z_{12}),\\
\mathcal{F}[J_{11}(\beta_1,\beta_{11'})\varphi_1(\beta_i)] &= \int d\alpha\ \chi_1(\alpha)\delta(z_{11'}-\alpha z_{12})\ \delta(z_{22'}),\\
\mathcal{F}[J_{11}(\beta_1,-\beta_2)\varphi_0(\beta_i)] &= \int d\alpha\ \ \chi_0(\alpha)\delta(z_{11'}-\alpha z_{12})\ \delta(z_{22'}+(1-\alpha)z_{12}),\\
\mathcal{F}[\delta(\beta_1,-\beta_2)] &= \delta(z_{11'})\ \delta(z_{22'}).
\end{aligned}
\tag{8.6}
$$

β dependent factors multiplying the functions J_{11} result in some additional functions in the intergrand of the Fourier transformed Feyman parameter representations:

$$
\begin{aligned}
\varphi_1(\beta_i) = 1 &\quad\rightarrow\quad \chi_1(\alpha) = 1,\\
\varphi_1(\beta_i) = \frac{\beta_1}{\beta_{1'}} &\quad\rightarrow\quad \chi_1(\alpha) = 1-\alpha,\\
\varphi_1(\beta_i) = \frac{\beta_1}{\beta_{11'}} &\quad\rightarrow\quad \chi_1(\alpha) = -\frac{1-\alpha}{\alpha},\\
\varphi_1(\beta_i) = \frac{\beta_1^2}{\beta_{1'}\beta_{11'}} &\quad\rightarrow\quad \chi_1(\alpha) = -\frac{(1-\alpha)^2}{\alpha},\\
\varphi_0(\beta_i) = 1 &\quad\rightarrow\quad \chi_0(\alpha) = 1,\\
\varphi_0(\beta_i) = \frac{\beta_1}{\beta_1+\beta_2} &\quad\rightarrow\quad \chi_0(\alpha) = 1-\alpha,\\
\varphi_0(\beta_i) = \frac{\beta_2}{\beta_1+\beta_2} &\quad\rightarrow\quad \chi_0(\alpha) = \alpha,\\
\varphi_0(\beta_i) = \frac{\beta_1\beta_2}{(\beta_1+\beta_2)^2} &\quad\rightarrow\quad \chi_0(\alpha) = \alpha(1-\alpha),
\end{aligned}
\tag{8.7}
$$

9 Appendix B

In the case of antiparallel helicity gluons the calculation of the effective two-parton interaction results instead of (3.8) in

$$
\begin{aligned}
\left(\frac{\partial_{1'}^2 \partial_{2'}^2 + \partial_1^2 \partial_2^2}{(\partial_1 + \partial_{1'})^2} \ \tilde{J}_{111} + \frac{(\partial_{1'}\partial_{2'} + \partial_1 \partial_2)\partial_1 \partial_2}{(\partial_1 + \partial_{1'})^2} \ \tilde{J}_0 \right) (A_1^* T^a A_{1'}) \ (A_2 T^a A_{2'}^*) \\
- \frac{(\partial_1 \partial_{1'} + \partial_2 \partial_{2'})\partial_1 \partial_2}{(\partial_1 + \partial_2)^2} \ \tilde{J}_0 \ \ (A_1^* T^a A_2) \ (A_{1'} T^a A_{2'}^*) \\
+ (\partial_1 + \partial_{1'})^2 \ \ \tilde{J}_{111} \ \ (A_1^* T^a A_{1'}^*) \ (A_2 T^a A_{2'})
\end{aligned}
\tag{9.1}
$$

Here the self-energy contributions are not included yet. We separate in the first bracket a term equal to the result for the parallel helicity case (3.8). Adding now the self-energy contribution (3.11) results in the replacement of the first bracket by

$$
2\partial_{1'}\partial_{2'}[\tilde{J}_{11'} + w_g^{(0)} \delta^{(2)}] - (\partial_{1'} - \partial_1)(\partial_{2'} - \partial_2) \ \tilde{J}_{111} + \partial_1 \partial_2 \tilde{J}_0, \tag{9.2}
$$

where now each term represents a regular operator.

The last two relations given in (8.3) for the J integrals Fourier transformed to light ray variables can be used now to substitute $(\partial_{1'} - \partial_1)(\partial_{2'} - \partial_2) \ \tilde{J}_{111}$ and $(\partial_{1'} + \partial_1)^2 \ \tilde{J}_{111}$ by $\tilde{J}_{112}$ and $\tilde{J}_{221}$ plus remainders involving $\tilde{J}_0$. This results in

$$
\begin{aligned}
\left(2[\tilde{J}_{11'} + w_g^{(0)} \delta^{(2)}] + \tilde{J}_{221} - 2\tilde{J}_{112} \right) \ (A_1^* T^a \partial A_{1'}) \ (A_2 T^a \partial A_{2'}^*) \\
+ \tilde{J}_{221} \ \ (A_1^* T^a \partial A_{1'}^*) \ (A_2 T^a \partial A_{2'}) \\
+ \frac{\partial_1 \partial_2}{(\partial_1 + \partial_2)^2} \tilde{J}_0 \ \ \{ -(\partial_1 \partial_{2'} + \partial_2 \partial_{1'})(A_1^* T^a A_{1'}^*) \ (A_2 T^a A_{2'}) \\
+ (\partial_1 \partial_{1'} + \partial_2 \partial_{2'})[(A_1^* T^a A_{1'}) \ (A_2 T^a A_{2'}^*) - (A_1^* T^a A_2) \ (A_{1'} T^a A_{2'}^*)] \}
\end{aligned}
\tag{9.3}
$$

Due to the commutation relation of the generators T^a we have for the gauge group brackets (2.5)

$$
(A_1^* T^a A_{1'}) \ (A_2 T^a A_{2'}^*) - (A_1^* T^a A_2) \ (A_{1'} T^a A_{2'}^*) = (A_1^* T^a A_{2'}^*) \ (A_2 T^a A_{1'}) \tag{9.4}
$$

Thus the last term in the brackets multiplying $\tilde{J}_0$ is equal to the first one up to the sign and the exchange of $1'$ and $2'$. We remember that the above expressions are understood as integrated over the light ray positions $1, 2, 1', 2'$. Therefore the exchange of $1'$ and $2'$ is just a substitution of integration variables. We conclude that the contribution involving $\tilde{J}_0$ cancels and arrive at the result (5.6) up to normalization.